



Recursion

Binary Search Algorithm

Lecture Contents

- Linear Search
- Binary Search
 - Details
 - Algorithm
- Big O Complexity (just a mention)

Linear Search

0	1	2	3	3	3	5	7	8	8	9
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- Linear search is “brute force”
- Just “walk” through the array until we find the target
- Simple, easy... and maybe best if the array is not already sorted

```
public static int search(int[] a, int target) {  
    for (int i=0; i<a.length; i++) {  
        if (a[i] == target)  
            return i;  
    }  
    return -1;  
}
```


Binary Search

- Details
 - Requires a sorted list
 - Very efficient
 - Divide and conquer

Binary Search


- Find where the value 5 is in our array.

0	1	2	3	3	3	5	7	8	8	9
---	---	---	---	---	---	---	---	---	---	---

Binary Search

- Find where the value 5 is in our array.
- Algorithm:
 - Find the middle
 - how?

middle




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Binary Search

- Find where the value 5 is in our array.
- Algorithm:
 - Find the middle
 - $a.length / 2$
 -

middle




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Binary Search

- Find where the value 5 is in our array.
- Algorithm:
 - Find the middle
 - $a.length / 2$
 - Is the middle our target?

middle




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Binary Search

- Find where the value 5 is in our array.
- Algorithm:
 - Find the middle
 - $a.length / 2$
 - Is the middle our target?
 - No, so where do we search?

middle




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Binary Search

- Find where the value 5 is in our array.

- Algorithm:

- Find the middle
 - $a.length / 2$
- Is the middle our target?
 - No, so where do we search?
- Hmm... it seems we're going to need to keep track of the bounds of the array that still needs to be searched.

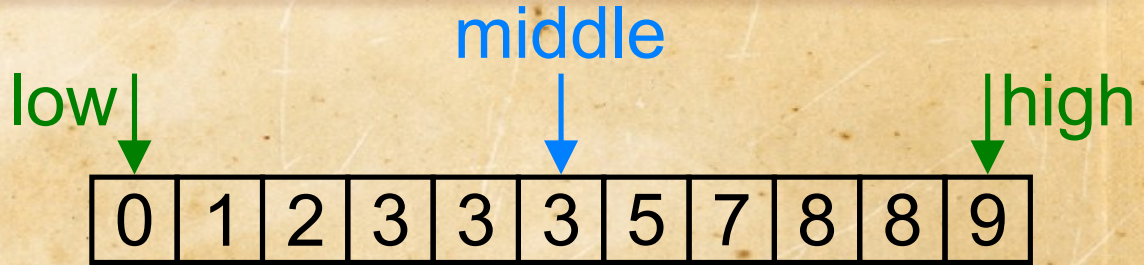
middle



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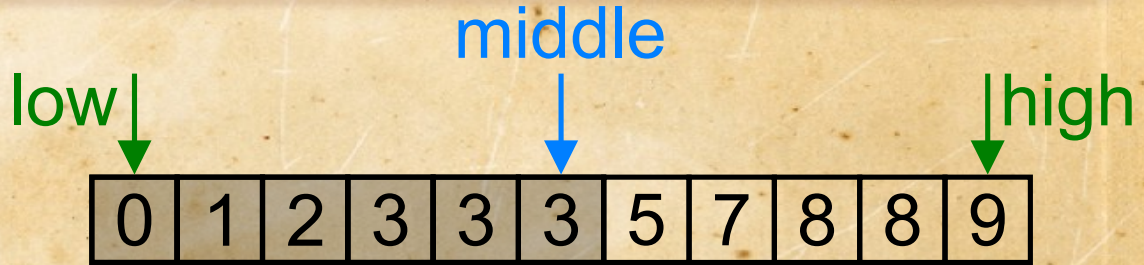
Binary Search

- Find where the value 5 is.
- Algorithm:
 - Find the middle
 - $a.length / 2$
 - Is the middle is the target?



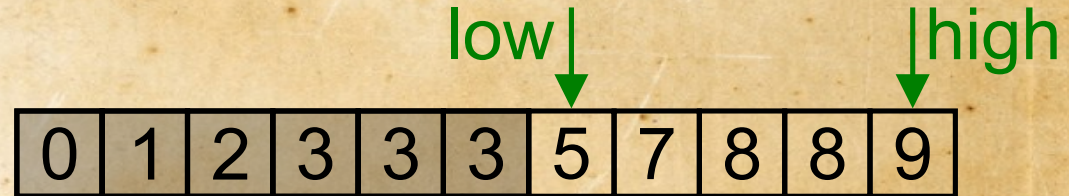
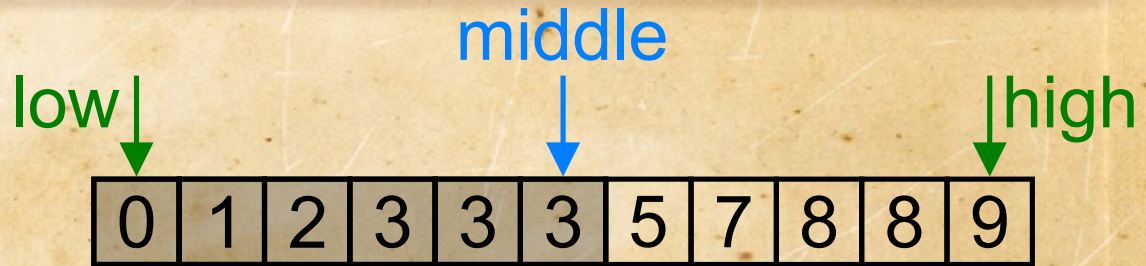
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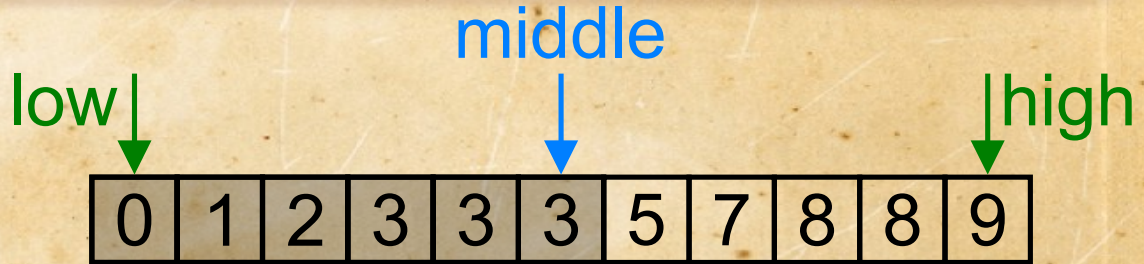


Binary Search

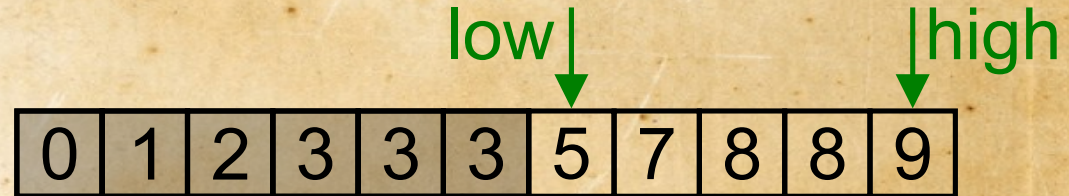
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- Algorithm:

- Find the middle
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- Is the middle is the target?



← **Now this ain't gonna work!**



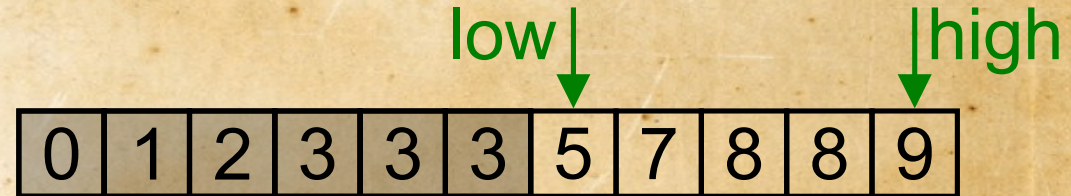
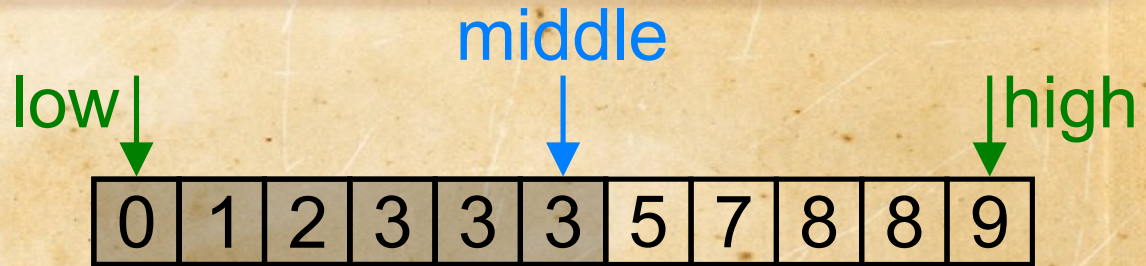
Binary Search

- Find where the value 5 is.
- Algorithm:

- Find the middle

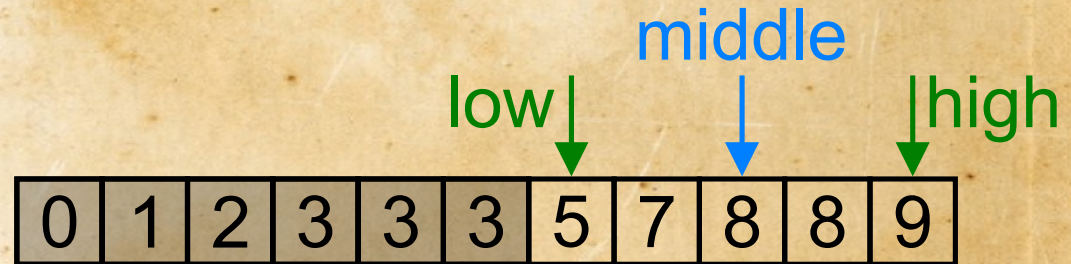
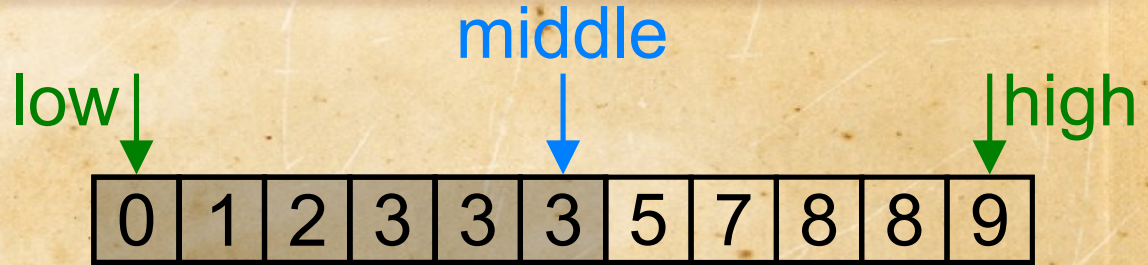
- $(\text{high} + \text{low}) / 2$ ← **Now this will work!**

- Is the middle is the target?



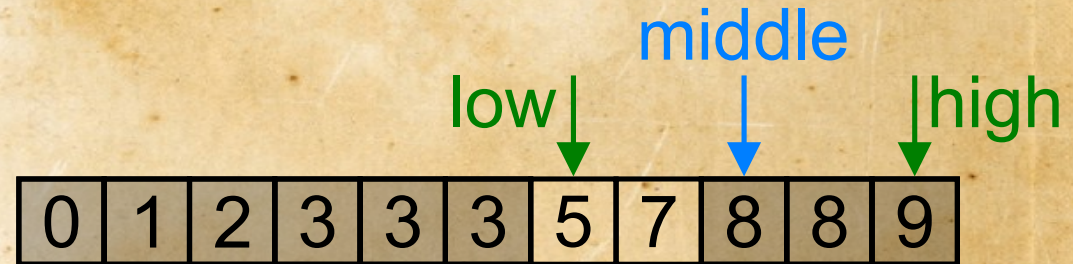
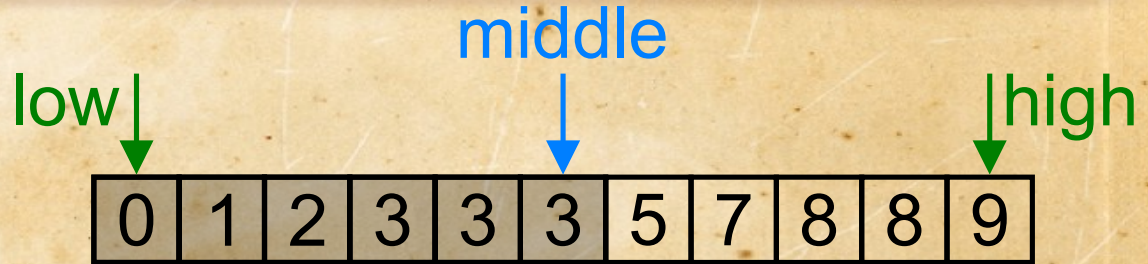
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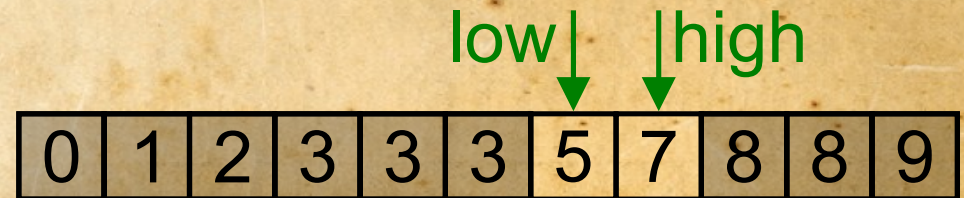
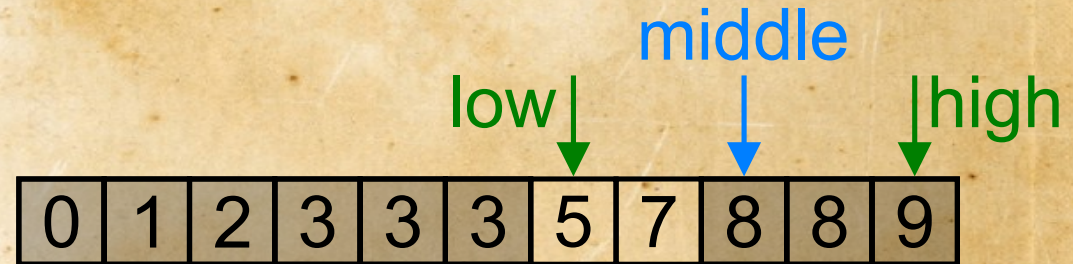
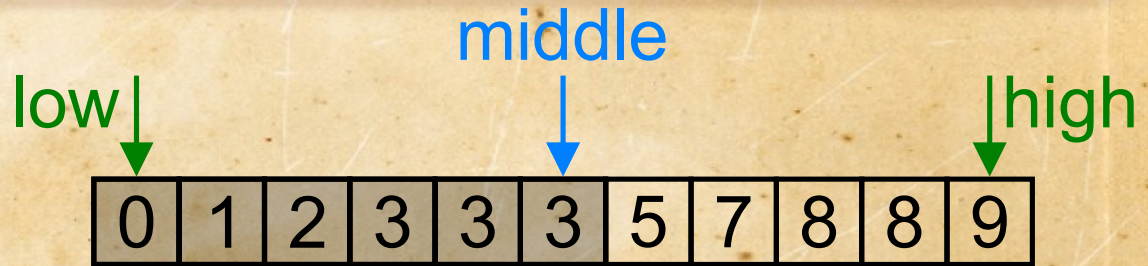
Binary Search

- Find where the value 5 is.
- Algorithm:
 - Find the middle
 - Is the middle is the target?
 - Nope; this time we search the lower half



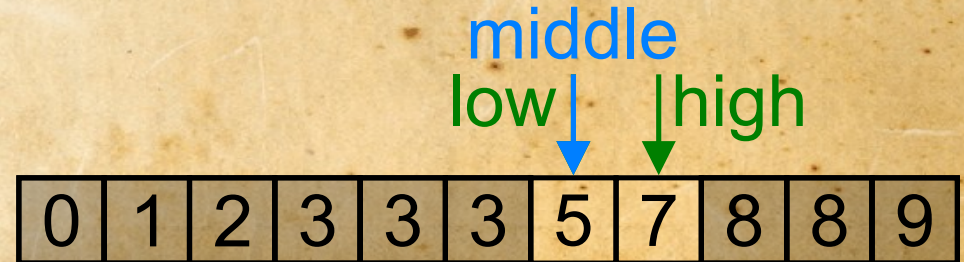
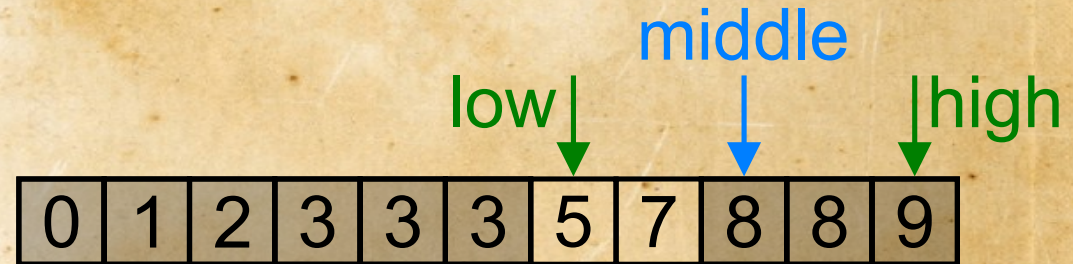
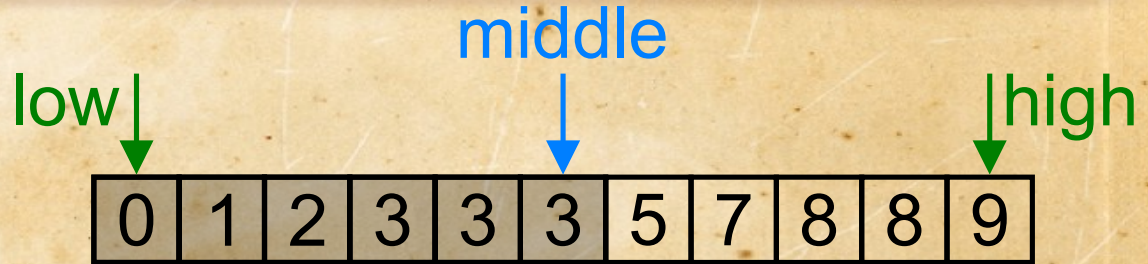
Binary Search

- Find where the value 5 is.
- Algorithm:
 - Find the middle
 - Is the middle is the target?
 - Nope; this time we search the lower half
 - Where's the middle of this one?



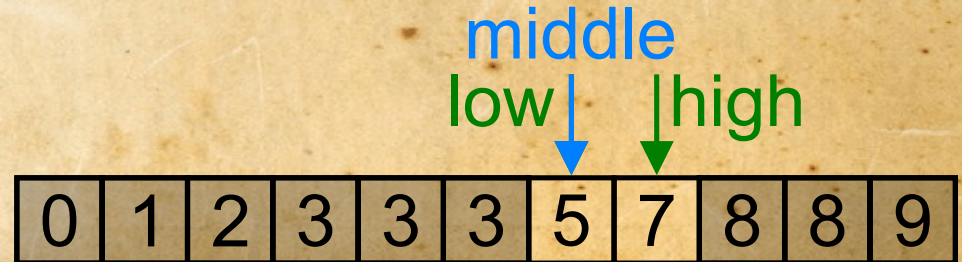
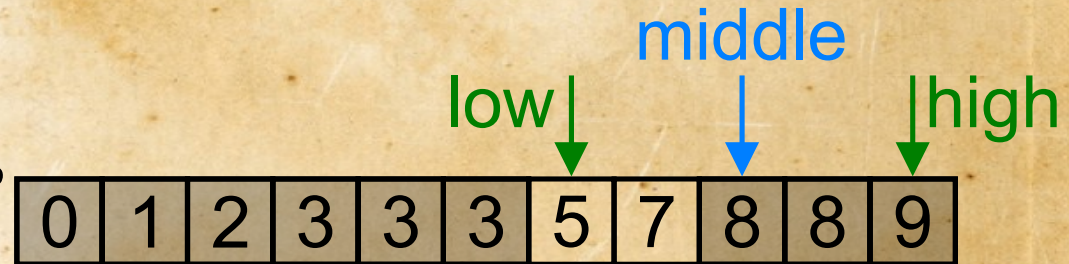
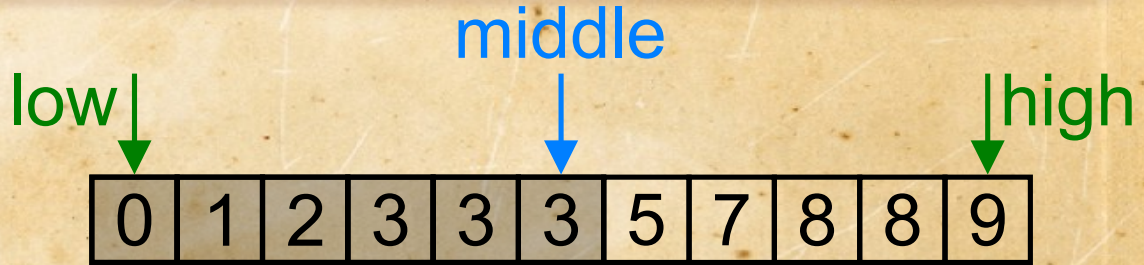
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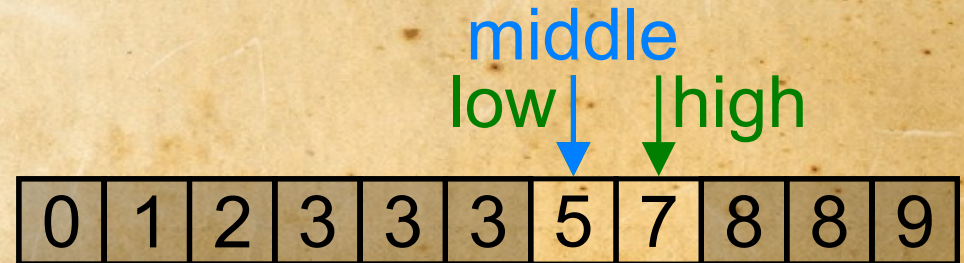
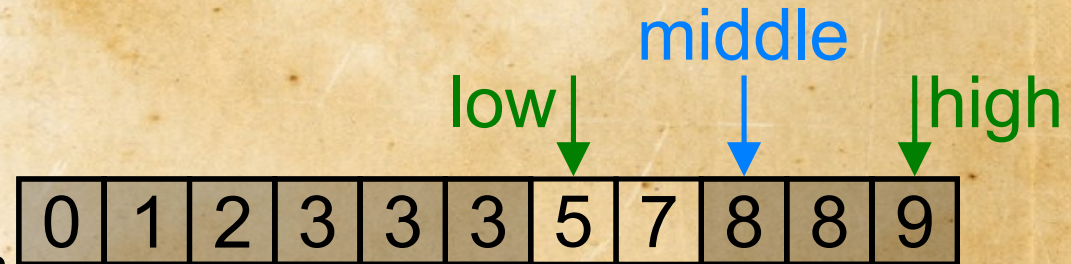
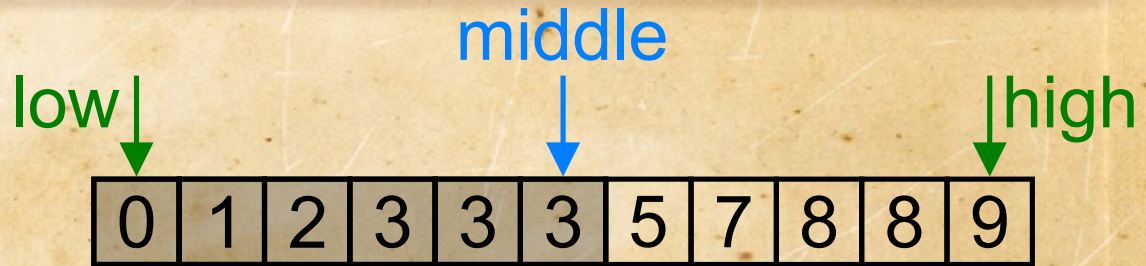
Binary Search

- Find where the value 5 is.
- Algorithm:
 - Find the middle
 - Is the middle is the target?
 - If yes, return index
 - Is the middle greater than target?
 - If yes, search lower
 - Is the middle lesser than target?
 - If yes, search upper



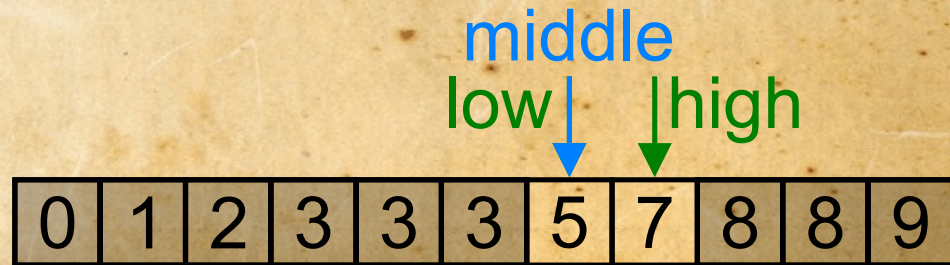
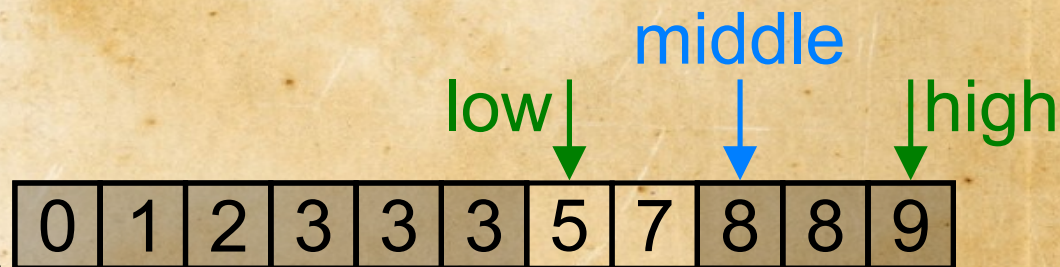
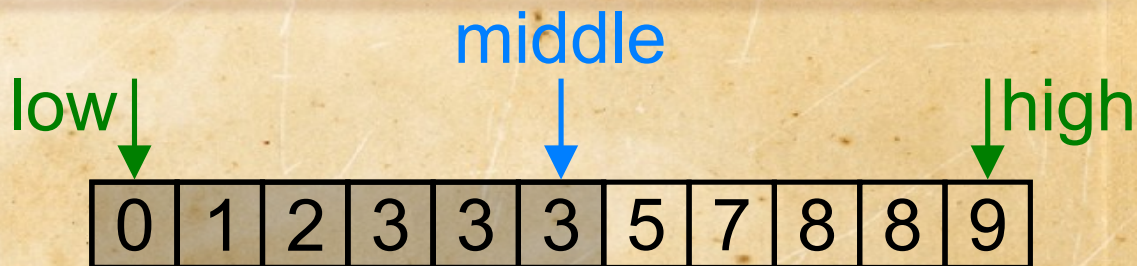
Binary Search

- Find where the value 5 is.
- Algorithm:
 - **If no array left, return -1**
 - Find the middle
 - Is the middle is the target?
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Binary Search

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- What's the longest a *linear search* would take to search an array with *n* elements in it?

Binary Search

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- What's the longest a linear search would take to search an array with n elements in it?
 - n comparisons

Binary Search

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- What's the longest a linear search would take to search an array with n elements in it?
 - n comparisons
- What's the longest a binary search would take to search a sorted array with n elements in it?

Binary Search

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- What's the longest a linear search would take to search an array with n elements in it?
 - n comparisons
- What's the longest a binary search would take to search a sorted array with n elements in it?
 - $\log n$ comparisons

Binary Search

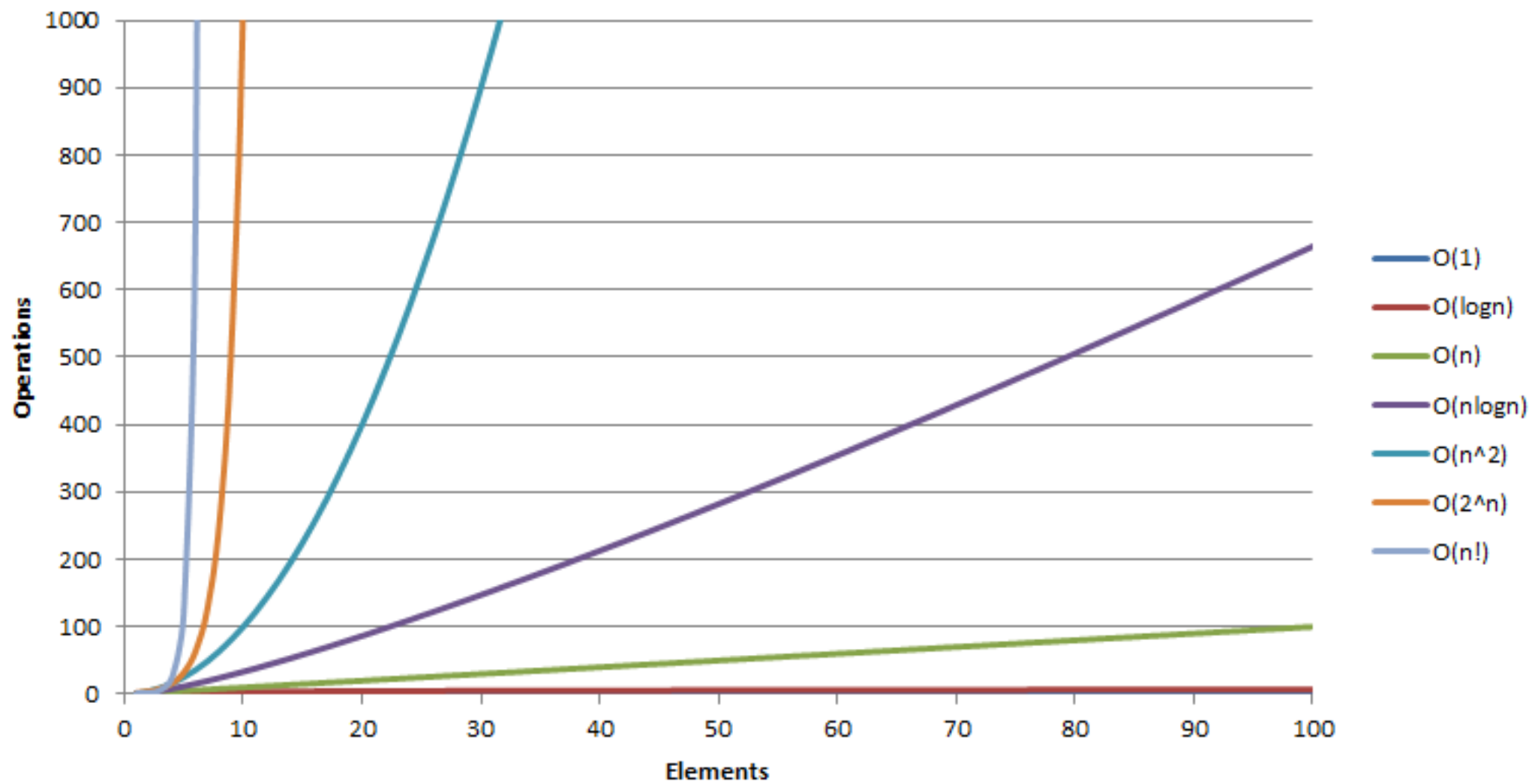
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- What's the longest a linear search would take to search an array with n elements in it?
 - n comparisons
- What's the longest a binary search would take to search a sorted array with n elements in it?
 - $\log n$ comparisons
- In computer science, algorithm efficiency is discussed using "*Big O Notation*"

Big O Notation

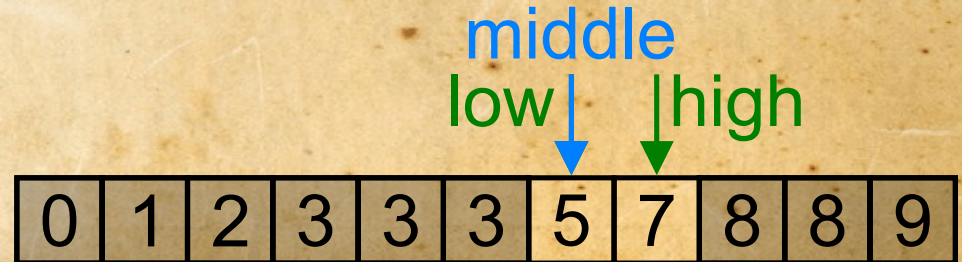
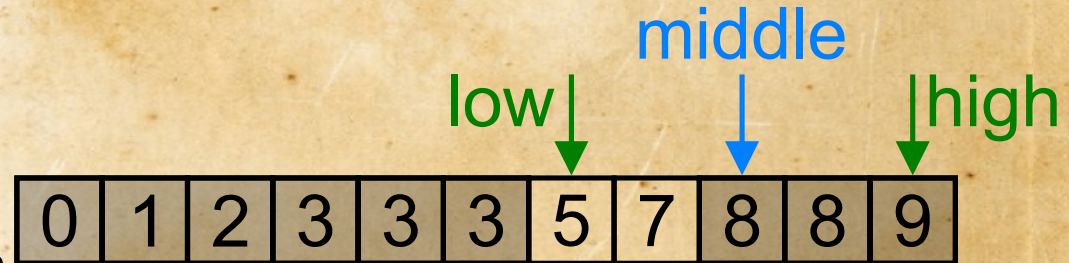
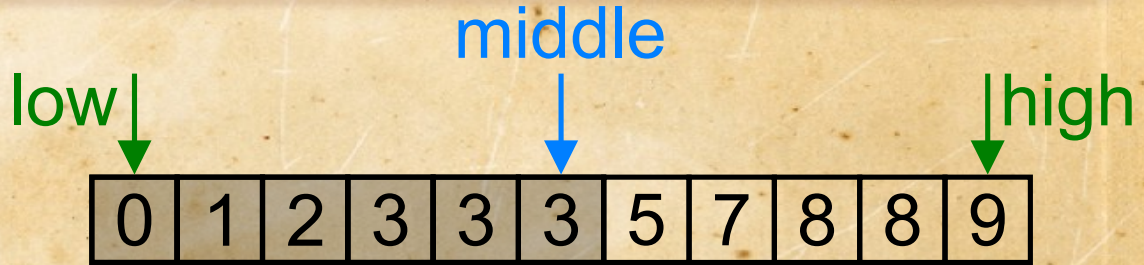
$O(n)$	linear	Finding an item in an unsorted list or in an unsorted array; adding two n -bit integers by ripple carry
$O(n \log^* n)$	n log-star n	Performing triangulation of a simple polygon using Seidel's algorithm , or the union-find algorithm . Note that $\log^*(n) = \begin{cases} 0, & \text{if } n \leq 1 \\ 1 + \log^*(\log n), & \text{if } n > 1 \end{cases}$
$O(n \log n) = O(\log n!)$	linearithmic, loglinear, quasilinear, or " $n \log n$ "	Performing a fast Fourier transform ; fastest possible comparison sort ; heapsort and merge sort
$O(n^2)$	quadratic	Multiplying two n -digit numbers by schoolbook multiplication ; simple sorting algorithms, such as bubble sort , selection sort and insertion sort ; (worst-case) bound on some usually faster sorting algorithms such as quicksort , Shellsort , and tree sort

Big-O Complexity



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